Global Positioning System: Finding a location.

A signal is being transmitted from your location to three cell towers that are around your location. The satellites determine the distance you are from the towers. To make this activity easier, we will place the towers at coordinate points on the grid.

Tower 1 is located at (9, 3); Tower 2 at (-2, 4); and Tower 3 at (7, -7). Draw a coordinate grid and place the towers on the grid.

The signal being transmitted from your location is (4, 3).

Find the distance between your signal and the three towers.

Distance between signal and tower 1 _____

$$d = \sqrt{\left((9-4)^2 + (3--3)^2\right)} = \sqrt{25+36} = \sqrt{61} \cong 7.8 miles$$

Distance between signal and tower 2 _____

$$d = \sqrt{\left((-2--4)^2 + (4--3)^2\right)} = \sqrt{36+49} = \sqrt{85} \cong 9.2 miles$$

Distance between signal and tower 3 _____

$$d = \sqrt{\left((7-4)^2 + (-7--3)^2\right)} = \sqrt{9+16} = \sqrt{25} = 5 miles$$

Draw the triangle that the three towers create on the grid. GPS systems use triangulation to locate the specific latitude and longitude of a signal.

The three towers transmit in a circular fashion. Use will uses the distance that your signal gives off to the three different towers as the information that we need to write equations for three circles.

Equations for a circle need three pieces of information. 1) the equation for a circle, 2) the center of the circle, and 3) the radius of the circle.

The equation for a circle at the origin is $x^2 + y^2 = r^2$

An equation for a circle at any other point (h, k) is: $(x - h)^2 + (y - k)^2 = r^2$

List the three equations for the towers using their given center and distance as the radius.

Tower 1: _____

$\boldsymbol{(x-9)^2 + (y-3)^2 = 61}$

Tower 2: _____

$\boldsymbol{(x+2)^2 + (y-4)^2 = 85}$

Tower 3: _____

$\boldsymbol{(x-7)^2 + (y+7)^2 = 25}$

On the grid, draw the three circles and find the places where two circles intersect.

Math behind the GPS locator:

The equations for a circle are in standard form. The equations need to be expanded in order for us to solve the system of equations.

Remember, when expanding a polynomial, you should get three terms for each polynomial.

Hint: $(x + 3)^2 = (x + 3)(x + 3) = x^2 + 6x + 9$

Expand each equation and write the expanded form:

Tower 1: _____

$\boldsymbol{x^2 - 18x + 81 + y^2 - 6y + 9 = 61}$

Tower 2: _____

$\boldsymbol{x^2 + 4x + 4 + y^2 - 8y + 16 = 85}$

Tower 3: _____

$$x^2 - 14x + 49 + y^2 + 14y + 49 = 25$$

To solve a system of equations, take two equations at a time and find the difference between them. When you find the difference, simplify until you get a linear equation in slope-intercept form.

Take Tower 1 – Tower 2;

$$x^2 - 18x + 81 + y^2 - 6y + 9 = 61$$

$$-(x^2 + 4x + 4 + y^2 - 8y + 16 = 85)$$

$$-22c + 77 + 2y - 7 = -24$$

$$2y = 22x - 94$$

$$y = 11x - 47$$ Linear Equation: y=11x-47

Take Tower 1 – Tower 3

$$x^2 - 18x + 81 + y^2 - 6y + 9 = 61$$

$$-\ (x^2 - 14x + 49 + y^2 + 14y + 49 = 25)$$

$$-4x + 32 - 20y - 40 = 36$$

$$-20y = 4x + 44$$

$$y = -\frac{1}{5}x - \frac{11}{5}$$ Linear Equation: $y = -\frac{1}{5}x - \frac{11}{5}$

Using the two linear equations, use substitution and solve for both x and y.

$$11x - 47 = -\frac{1}{5}x - \frac{11}{5}$$

$$\frac{55}{5}x = \frac{224}{5}$$

$$x = 4$$

$$y = 11(4) - 47$$

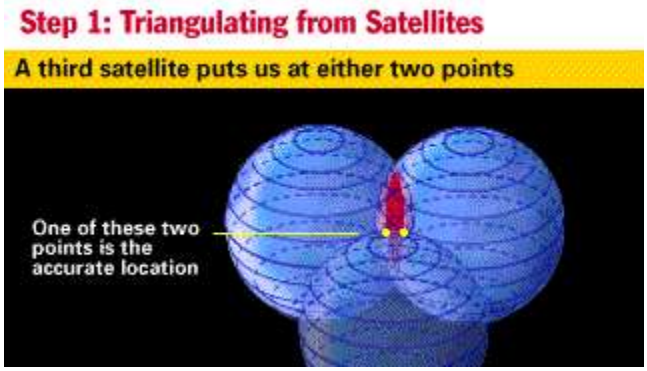
$$y = 3$$

Point of intersection: ___(4 , 3)____

We did this activity to understand the process that GPS goes through to locate a signal.

There is one big difference; we used the coordinate plane, GPS uses three dimensional space. Below is a table that shows the differences between our model and the actual GPS system.

GPS	Our example
Uses Satellites	Used cell towers
Uses spheres	Used circles
Uses longitude and latitude coordinate system	Used ordered pairs on coordinate grid
Uses four dimensions for measurements	Used two dimensions
Uses triangulation in 3 space	Used triangulation in 2 space



http://www.trimble.com/gps/howgps-triangulating.shtml

Precision in GPS systems

Earlier we learned that each Longitude and Latitude degree is approximately equal to 69 miles. If the address only contains degrees, your can locate what you are looking for within 69 miles^2 or a total of 4,761 miles of land. If the address contains degrees and minutes, you narrow the search area to about 1.15 miles^2. (Each minute divides up the degree up into 60 sections.) If the address contains degrees, minutes and seconds, the location can be found within 33.3 yards^2 of the object. If the seconds are expanded to two or more decimal places, the precision will be closer than 2 feet ^{2} from the object.